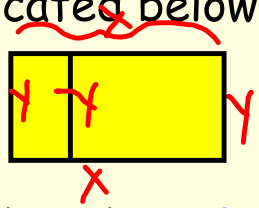
Precalc Warm Up # 8-2

1. A person wants to build an enclosure for their garden. They have 400 feet of fence to enclose two areas as indicated below. What dimensions maximize the area?



$400 = 2x + 3y$

$\frac{400-2x}{3} = y$

$A = xy$

$A = x\left(\frac{400-2x}{3}\right)$

$x = 0$ $\quad x = 200$

$V(100, y)$

$400 = 2(100) + 3y$

$200 = 3y$

$y = 66\frac{2}{3}$

2. Sketch $y = -2x^2(x-3)^3(x+2)(x-6)^4$

a. Describe end behavior

b. Check with grapher

c. How many turning points does it have?

d. At most, how many turning points could a 10th degree equation have?

e. How many zeros does it have?

f. At most, how many zeros could a 10th degree equation have?

2. Sketch $y = -2x^2(x-3)^3(x+2)(x-6)^4$

a. Describe end behavior $\longrightarrow$ as $x \to \infty$, $f(x) \to -\infty$

as $x \to -\infty$, $f(x) \to -\infty$

b. Check with grapher

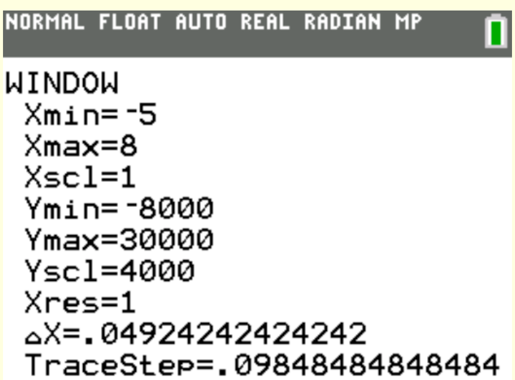
c. How many turning points does it have? 5

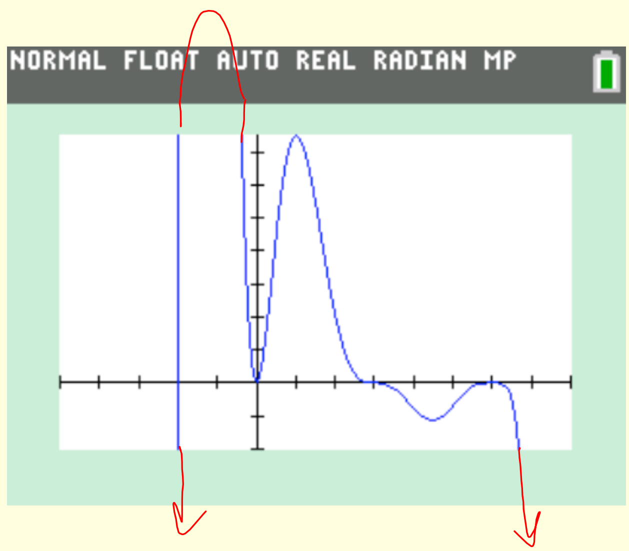
d. At most, how many turning points could a 10th degree equation have? 9

$n-1$ (max # of turns)

e. How many zeros does it have? 4

f. At most, how many zeros does a 10th degree equation have? 10

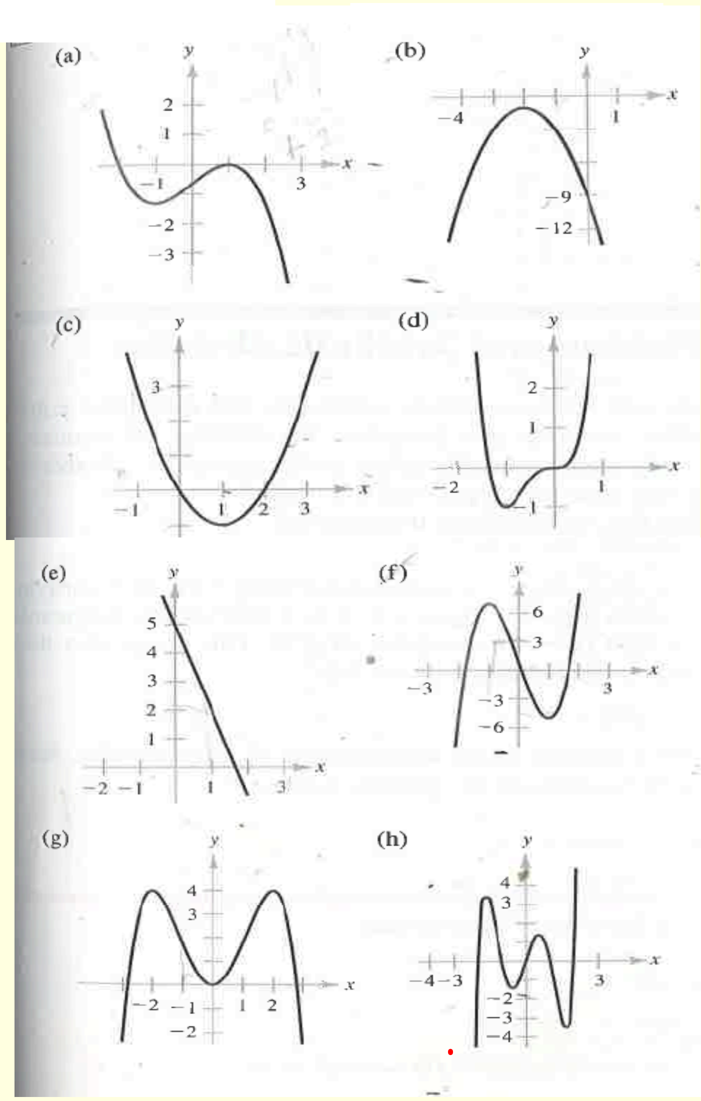




HW Questions: p. 191
Matching

1. $f(x) = -3x + 5$
3. $f(x) = -2x^2 - 8x - 9$
5. $f(x) = -\frac{1}{3}x^3 + x - \frac{2}{3}$
7. $\boxed{7.}$ $f(x) = 3x^4 + 4x^3$

2. $f(x) = x^2 - 2x$
4. $f(x) = 3x^3 - 9x + 1$
6. $f(x) = -\frac{1}{4}x^4 + 2x^2$
8. $f(x) = x^5 - 5x^3 + 4x$



In Exercises 9–18, determine the right-hand and left-hand behavior of the graph of the polynomial function.

11. $g(x) = 5 - \frac{7}{2}x - 3x^2$

15. $f(x) = 6 - 2x + 4x^2 - 5x^3$

In Exercises 19–34, find all the real zeros of the polynomial function.

21. $h(t) = t^2 - 6t + 9$

25. $f(x) = 3x^2 - 12x + 3$

$0 = 3(x^2 - 4x + 1)$

33. $f(x) = 5x^4 + 15x^2 + 10$

$0 = 5(x^4 + 3x^2 + 2)$

$0 = 5(x^2 + 2)(x^2 + 1)$

$x^2 + 2 = 0$

$\sqrt{x^2} = \sqrt{-2}$

No real zeros

29. $g(t) = \frac{1}{2}t^4 - \frac{1}{2}$

$= \frac{1}{2}(t^4 - 1)$

$= \frac{1}{2}(t^2 - 1)(t^2 + 1)$

$0 = \frac{1}{2}(t + 1)(t - 1)(t^2 + 1)$

In Exercises 35–44, find a polynomial function that has the given zeros.

$\boxed{35.}$ 0, 10

$\boxed{41.}$ 4, −3, 3, 0

43. $1 + \sqrt{3}$, $1 - \sqrt{3}$

$f(x) = x(x-4)(x+3)(x-3)$

$x(x-4)(x^2-9)$

$(x - (\text{the zero}))$

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In Exercises 35–44, find a polynomial function that has the given zeros.

35. 0, 10

$f(x) = (x - 0)(x - 10)$

$f(x) = x(x - 10)$

$f(x) = x^2 - 10x$

41. 4, −3, 3, 0

43. $1 + \sqrt{3}$, $1 - \sqrt{3}$

$f(x) = (x - (1 + \sqrt{3}))(x - (1 - \sqrt{3}))$

$f(x) = ((x - 1) - \sqrt{3})((x - 1) + \sqrt{3})$

Notice sum & difference pattern gives you the difference of squares.

$f(x) = (x - 1)^2 - (\sqrt{3})^2$

$f(x) = x^2 - 2x + 1 - 3$

$\boxed{f(x) = x^2 - 2x - 2}$

In Exercises 47–58, sketch the graph of the given function.

$\boxed{47.}$ $f(x) = -\frac{3}{2}$

$\boxed{51.}$ $f(x) = x^3 - 3x^2$

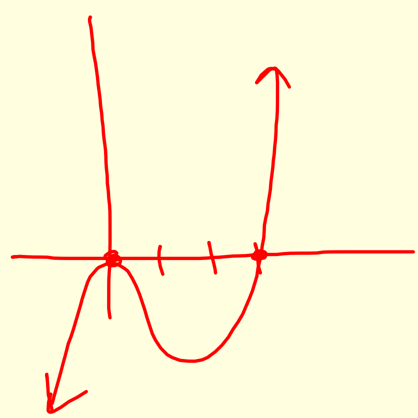
$f(x) = x^2(x-3)$

zeros @ 0 & 3

repeat zero touches x-axis without passing through.

Opposite end behavior.
Rises Rt.

$\boxed{55.}$ $g(t) = -\frac{1}{4}(t-2)^2(t+2)^2$



Use long division to divide the dividend 30489 by the divisor 22

$$
\begin{array}{r}
1385 \\
22 \,\overline{)\,30489} \\
-22 \\
\hline
84 \\
-66 \\
\hline
188 \\
-176 \\
\hline
129 \\
-110 \\
\hline
19
\end{array}
$$

19 remainder

Quotient

$1385 + \boxed{\frac{19}{22}}$ remainder

Divide the polynomial $f(x) = 8x^3 - 20x^2 + 3x - 1$ by $x-5$.

$$
\begin{array}{r}
8x^2 + 20x + 103 \\
x-5 \overline{\big) \; 8x^3 - 20x^2 + 3x - 1} \\
+ \; (-8x^3 + 40x^2) \\
\hline
20x^2 + 3x \\
- \; (20x^2 - 100x) \\
\hline
103x - 1 \\
+ \; (-103x + 515) \\
\hline
514
\end{array}
$$

remainder: 514

Quotient

$$8x^2 + 20x + 103 + \frac{514}{x-5}$$

Divide $f(x) = 8x^3 - 20x^2 + 3x - 1$ by $x - 5$.

$x-5=0$ $\boxed{x=5}$ divisor

Synthetic division: A short cut when dividing by a linear factor $(x - k)$

$$
\begin{array}{c|cccc}
5 & 8 & -20 & 3 & -1 \\
 & \downarrow & 40 & 100 & 515 \\
\hline
 & 8 & 20 & 103 & 514
\end{array}
$$

remainder

Quotient $\boxed{8x^2 + 20x + 103 + \frac{514}{x-5}}$

Divide $f(x) = 8x^3 - 20x^2 + 3x - 1$ by $x - 5$.

Is $x - 5$ a factor of $f(x)$?

NO, because there is a remainder

$$\frac{8x^3-20x^2+3x-1}{x-5} = 8x^2+20x+103+\frac{514}{x-5}$$

Synthetic division is easier, but when the divisor is not linear you MUST use long division:

Ex: Divide $2x^4 + 3x - 4$ by $x^2 - x + 5$

* Notice the place holders for the missing powers.

$$
\begin{array}{r}
2x^2 + 2x - 8 \\
x^2 - x + 5 \,\big)\, \overline{2x^4 + 0x^3 + 0x^2 + 3x - 4} \\
+\,(-2x^4 + 2x^3 - 10x^2) \qquad\quad \\
\hline
2x^3 - 10x^2 + 3x \qquad \\
+\,(-2x^3 + 2x^2 - 10x) \qquad \\
\hline
-8x^2 - 7x - 4 \\
-\,(-8x^2 + 8x - 40) \\
\hline
-15x + 36
\end{array}
$$

remainder $-15x+36$

Answer:
$$2x^2 + 2x - 8 + \frac{-15x+36}{x^2-x+5}$$

Divide $f(x) = x^4 - 3x^3 + 10x^2 - 4x + 8$ by $x - 3$.

$$\begin{array}{r|rrrrr} 3 & 1 & -3 & 10 & -4 & 8 \\ & \downarrow & 3 & 0 & 30 & 78 \\ \hline & 1 & 0 & 10 & 26 & 86 \end{array}$$

remainder

Quotient: $x^3 + 10x + 26 + \frac{86}{x-3}$

What is the zero of (x - 3)?

$x - 3 = 0$

$\boxed{x = 3}$

Find $f(3) = (3)^4 - 3(3)^3 + 10(3)^2 - 4(3) + 8$

$\cancel{81} - \cancel{81} + 90 - 12 + 8 = \boxed{86}$

What is the remainder to the original problem?

$f(3) = 86$

REMAINDER THEOREM:

If f(x) is divided by (x-k), then f(k) is the remainder r

when you do syn. ÷

$\boxed{f(k) = r}$

$f(x) = x^3 - 4x + 2$ Find f(-8) without a calculator.

-8	1	0	-4	2
	↓	-8	64	-480
	1	-8	60	-478

$f(-8) = -478$

REMAINDER THEOREM:

If f(x) is divided by (x-k), then $\boxed{f(k) = r}$

$x - k = 0$ $\quad$ $x = k$

Use the remainder theorem to evaluate:

$f(x) = 4x^5 + x^4 - 10x^3 + x^2 - 4$ at $x = -3$

$$\begin{array}{r|rrrrrr} -3 & 4 & 1 & -10 & 1 & 0 & -4 \\ & \downarrow & -12 & 33 & -69 & 204 & -612 \\ \hline & 4 & -11 & 23 & -68 & 204 & -616 \end{array}$$

remainder

$f(-3) = -616$

FACTOR THEOREM

$f(x)$ has a FACTOR $(x-k)$ if and only if $\boxed{f(k) = 0}$

remainder is 0
when you do ÷

HW PC Book:

p. 201 #5 - 31□, 33,
35 - 41□, 49bc